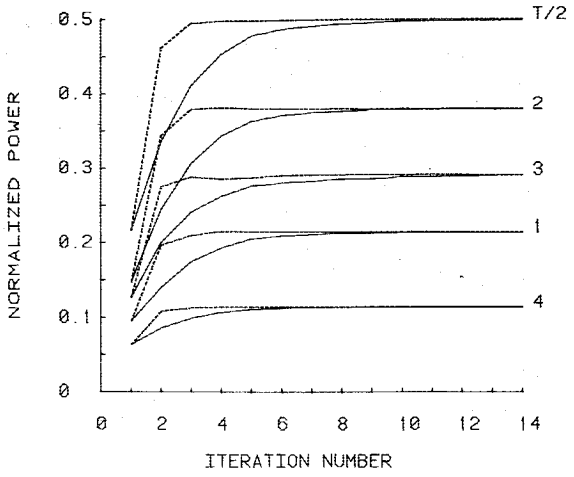

Fig. 2 Convergence history of output power in each vibrational band and their total normalized by the total output power for the 14th iteration using CONGI technique.--with CONGI,—without.

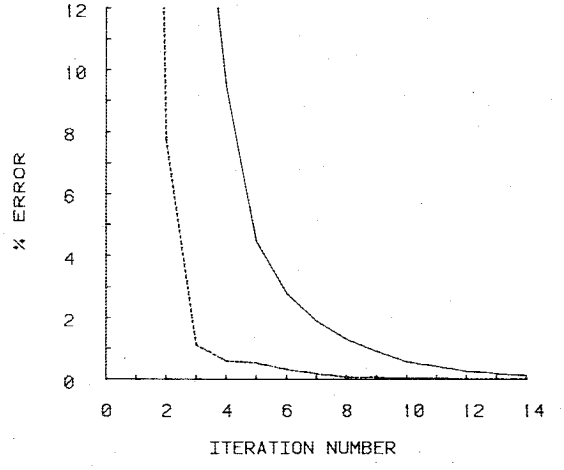

Fig. 3 History of percent error (deviation from the 14th iteration) of total output power.--with CONGI,—without.

module takes even less computer time than the PROP module and only about 1/100th of that of the GAIN module.

As shown schematically in Fig. 1, an inner loop is formed in which the CONGI-PROP cycle can be repeated any number of times before the propagated radiation fields are sent back to the GAIN-PROP (outer) loop. If the inner loop is repeated enough times, the radiation fields will converge. However, unless the flowfield has already converged, it is not very meaningful to continue the iteration of radiation fields in the inner loop until convergence. We have found that the radiation fields change relatively little after the first few iterations of the inner loop. Consequently, we have used the procedure of repeating the inner loop four times for every iteration of the outer loop. Since the CONGI module runs so much faster than the GAIN module, we have not attempted to determine the optimal number of inner-loop iterations. Actually, the optimum may very well depend on the particular case under study.

To demonstrate the effectiveness of the present technique in improving the convergence rate, the base case of Ref. 1 is recomputed with and without the CONGI-PROP inner loop. The starting radiation fields are uniform in the flow direction and the same for each of the 24 lasing transitions (six transitions for each of four vibrational bands). The convergence history of the total output power and the output power of the four individual bands, each normalized by the final value of the total output power, is shown in Fig. 2. Results obtained without using the CONGI-PROP inner loop are shown by solid lines and those with the inner loop are shown by dashed lines. The number of iterations shown corresponds to the number of the outer loops. The same starting conditions have been used for the two cases. The first iteration results of the two methods are identical because a GAIN-PROP loop is needed to determine $(gI)_{\text{previous}}$ for the CONGI-PROP loops. Figure 2 shows the effectiveness of the present technique in reducing the total number of iterations required. However, this is indicated more clearly in Fig. 3, which shows the percent error defined as the deviation of the total output power from that of the 14th iteration using the CONGI technique. For an accuracy of approximately 1%, only three iterations are needed using the CONGI technique. This saves about six iterations and two-thirds of the computer time required otherwise. Since one iteration takes approximately 15 min on a CDC 6500 computer, the cost saving obtained by using the CONGI technique is very substantial.

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Ignition of a Combustible Mixture by a Hot Particle

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Introduction

THE ignition of a combustible mixture by either hot or incendiary particles is a major cause of fires and explosions in mines, machine shops, and also in fuel tanks within both civilian and military vehicles. Whereas some experiments[1,2] have been conducted aiming to characterize the flammability of the mixture as a function of the ignition source, a reasonably rigorous theoretical analysis has not been attempted but is needed. We do so herein for the case when the ignition source is a hot, inert, particle; using matched asymptotic analysis in the realistic limit of large activation energies. It will be demonstrated that, through an appropriate transformation, the governing equations for the present problem can be cast to essentially the same forms as those for a similar problem[3] recently analyzed for the stagnation point ignition of a premixed combustible, such that existing solutions can be readily utilized. In particular, an explicit ignition criterion is derived.

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Governing Equations

We are interested in determining the ignitability of an infinite expanse of a combustible gaseous mixture at temperature T_∞ and consisting of known concentrations of an oxidizer, a fuel, and an inert, when a hot, spherical particle of uniform temperature T_W and radius r_W is placed within it.

First, it is reasonable to expect that the ignitability of the system is decided within the initial instant when the particle has just been introduced. This is because due to the relatively rapid rate of gas-phase transport, ignition occurs either immediately or not at all as the particle gradually cools down. Furthermore, due to the large thermal capacity of the hot particle, the particle temperature is not appreciably changed during this interval. Therefore T_W can be assumed to be a constant.

Assuming that the reactions between the fuel F and the oxidizer O leading to the formation of the product P can be represented by a one-step, overall irreversible reaction

$$\nu_O[O]+\nu_F[F]\rightarrow\nu_P[P] \tag{1}$$

which has a reaction rate proportional to

$$c_O^{a_O}c_F^{a_F}T^n e^{-E/R^oT}$$

the steady-state, spherically-symmetric, gas-phase diffusive-reactive heat and mass transport equations can be written as[4]

$$\mathcal{L}\{\tilde{Y}_O/L_O\}=\mathcal{L}\{\tilde{Y}_F/L_F\}=-\mathcal{L}\{\tilde{T}\}$$
$$=\delta\tilde{Y}_O^{a_O}\tilde{Y}_F^{a_F}\tilde{T}^{n-a_O-a_F}e^{-\tilde{T}_a/\tilde{T}} \tag{2}$$

where

$$\mathcal{L}=\frac{1}{\tilde{r}^2}\frac{\mathrm{d}}{\mathrm{d}\tilde{r}}\left(\tilde{r}^2\frac{\mathrm{d}}{\mathrm{d}\tilde{r}}\right)$$

$$\delta=\frac{B\nu_F W_F r_W^2\sigma^{a_O}}{(k/C_p)W_O^{a_O}W_F^{a_F}}\left(\frac{p\tilde{W}}{R^o}\right)^{a_O+a_F}\left(\frac{Q}{C_p}\right)^{n-a_O-a_F}$$

$\tilde{r}=r/r_W$, $\tilde{Y}_i=(\nu_F W_F)/(\nu_i W_i)Y_i$, $\tilde{T}=C_pT/Q$, $T_a=E/R^o$, r is the radial coordinate, c the molar concentration, Y the mass fraction, T the temperature, ρ the density, p the pressure, C_p the specific heat, k the thermal conductivity, Q the chemical heat release per unit mass of fuel reacted, R^o the universal gas constant, W the molecular weight, $\tilde{W}$ an average molecular weight, $\mathfrak{D}_i$ the mass diffusivity of i with respect to the rest of the species, B the pre-exponential factor for the reaction rate, E the activation energy, σ the stoichiometric oxidizer to fuel mass ratio, $L_i=k/(C_p\rho\mathfrak{D}_i)$ the Lewis number, and the subscripts W and ∞, respectively, designate the particle surface and the ambiance. It is of interest to note that the present formulation assumes constant, but nonunity, values of L_i.

Equation (2) is to be solved subject to the boundary conditions

$$\left(\frac{\mathrm{d}\tilde{Y}_i}{\mathrm{d}\tilde{r}}\right)_1=0 \quad \tilde{Y}_i(\infty)=\tilde{Y}_{i\infty} \quad i=O,F \tag{3}$$

$$\tilde{T}(1)=\tilde{T}_W \qquad \tilde{T}(\infty)=\tilde{T}_\infty \tag{4}$$

Analysis and Results

The production terms in the species equation in Eq. (2) can be eliminated through a linear combination of them with the energy equation,[4] yielding

$$\mathcal{L}\{\tilde{Y}_O/L_O+\tilde{T}\}=\mathcal{L}\{\tilde{Y}_F/L_F+\tilde{T}\}=0 \tag{5}$$

which can be readily integrated using Eqs. (3) and (4).

By further defining a new independent variable

$$\xi=1-\tilde{r}^{-1} \tag{6}$$

the final equations to be solved are

$$\frac{\mathrm{d}^2\tilde{T}}{\mathrm{d}\xi^2}=-\frac{\delta}{(1-\xi)^4}\tilde{Y}_O^{a_O}\tilde{Y}_F^{a_F}\tilde{T}^{(n-a_O-a_F)}e^{-\tilde{T}_a/\tilde{T}} \tag{7}$$

$$\tilde{T}(0)=\tilde{T}_W \quad \tilde{T}(1)=\tilde{T}_\infty \tag{8}$$

where

$$\tilde{Y}_i/L_i=\tilde{Y}_{i\infty}/L_i+\tilde{T}_\infty-(1-\xi)\left(\frac{\mathrm{d}\tilde{T}}{\mathrm{d}\xi}\right)_o-\tilde{T} \tag{9}$$

is the solution of Eq. (5).

Except for some nonessential differences, Eqs. (7-9) are almost identical to their counterparts for the stagnation point ignition problem analyzed in Ref. 3. Therefore, instead of carrying out the detailed analysis, we shall simply indicate the methodology and then state the final results.

The analysis capitalizes on the fact that the activation energy is usually a large number for chemical systems of interest to combustion. In the limit $\tilde{T}_a\rightarrow\infty$, the entire flowfield is frozen, such that Eqs. (7) and (8) yield a linear temperature profile. For large but finite values of $\tilde{T}_a$, a thin, "inner," diffusive-reactive region is expected to exist next to the particle surface which has the highest temperature. However, by moving slightly away from this region, the strong Arrhenius factor assures that even a small decrease in the gas temperature is sufficient to freeze the chemical reactions. This results in a much thicker, "outer," purely diffusive region. Hence the problem can best be analyzed using the inner-outer matched asymptotic expansion technique, with $\epsilon=\tilde{T}_W^2/\tilde{T}_a$ being the small parameter of expansion.

When analysis parallel to those detailed in Ref. 3 is carried out, it can be shown that by defining a reduced Damköhler number as

$$\Delta=\left(\frac{2\delta\epsilon}{\beta^2}\right)\left[\tilde{Y}_{O\infty}-\beta L_O\left(\frac{\mathrm{d}\theta}{\mathrm{d}\chi}\right)_o\right]^{a_O}\left[\tilde{Y}_{F\infty}-\beta L_F\left(\frac{\mathrm{d}\theta}{\mathrm{d}\chi}\right)_o\right]^{a_F}$$
$$\times\tilde{T}_W^{(n-a_O-a_F)}e^{-\tilde{T}_a/\tilde{T}_W} \tag{10}$$

where $\beta=\tilde{T}_W-\tilde{T}_\infty$ is a heat transfer parameter and

$$(\mathrm{d}\theta/\mathrm{d}\chi)_o-1=\mp(1-\Delta)^{1/2} \tag{11}$$

represents heat transfer at the surface, the asymptotic solutions of Eqs. (7) and (8) are identical to those of the stagnation point ignition problem.[3] In particular, it is found that Eqs. (7) and (8) have two solutions for $\Delta<1$ and no solution for $\Delta>1$. Hence, we have reproduced the lower half of the S-shaped response curve,[5] with

$$\Delta\geq 1 \tag{12}$$

being the criterion for systems in which ignition is expected to occur.

Furthermore, Eq. (11) shows that at the first instant when ignition is possible, $\Delta=1$, the heat transfer from the particle to the gas vanishes identically. This is physically realistic since there is now sufficient chemical heat release in the gas to sustain reactions. Indeed this adiabaticity limit has been intuitively used[6] in the past as the ignition criterion.

Acknowledgment

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Stagnation Point Boundary Layer on a Subliming Surface with Numerous Small Roughness Elements

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Introduction

LIBBY[1] presents an analysis of the laminar boundary layer at a two-dimensional stagnation point on a subliming surface which has a structure such that roughness elements result from the recession. The basic properties of the boundary layer depend in a fundamental way on a nondimensional parameter Γ which is proportional to n, the number of roughness elements per unit area. The problem is formulated so that as n and thus Γ increase, the extent of the roughness elements relative to the boundary-layer thickness decreases. When $\Gamma \sim \infty$ the boundary layer behaves as though slip occurs at the surface. Libby[1] demonstrates this result by means of an asymptotic analysis of a model problem obtained by neglecting the drag of the roughness elements; the outer solution is given by the usual stagnation point equations with modified boundary conditions at the wall and is shown to compare well with the complete solution of the equations involving roughness for $\Gamma = 10$. The purpose of this Note is to carry out the analysis of the complete asymptotic problem for constant density flow including the drag of the roughness elements.

Solution Methods

We begin by considering the streamwise momentum equation of Ref. 1 for the case of incompressible flow, namely

$$[(1-\alpha)F']' + (f-\alpha f_w)F' + (1-\alpha)(1-F^2) + \Gamma(2\alpha^{1/2}f_w - \alpha k)F = 0 \qquad (1)$$

where

$$F = f'(\eta)/(1-\alpha) \qquad (2)$$

$$\alpha = 1 \qquad \eta \le 0 \qquad (3a)$$

$$\alpha = (1-\Gamma\eta)^2 \qquad 0 \le \eta \le \Gamma^{-1} \qquad (3b)$$

$$\alpha = 0 \qquad \eta \ge \Gamma^{-1} \qquad (3c)$$

with the associated boundary conditions

$$\eta \to \infty, \quad f' = 1; \quad \eta = 0, \quad f - f_w = f' = 0 \qquad (4)$$

Primes denote differentiation with respect to the similarity variable η defined, along with the stream-function f, in standard form in Ref. 1. The specified parameter f_w represents the injection rate associated with sublimation, and k (in Libby's notation $= m_D \rho_s v_s / 2r$) is a measure of the drag on the fluid passing around the roughness elements. The quantity $\alpha(\eta)$ is the fraction of area covered by solid.

Now we define $\epsilon = \Gamma^{-1}$ and develop solutions in the limit $\epsilon \to 0$ for $f_w = 0(1)$ and $k = 0(1)$. In the outer region, defined by Eq. (3c), Eq. (1) and the first part of Eq. (4) become

$$f''' + ff'' + (1-f'^2) = 0; \quad \eta \to \infty, \quad f' = 1 \qquad (5)$$

If we assume that $f \sim f_0 + \epsilon f_1 + \ldots$, then

$$f_0''' + f_0 f_0'' + (1-f_0'^2) = 0; \quad \eta \to \infty, \quad f_0' = 1 \qquad (6)$$

$$f_1''' + f_0 f_1'' - \alpha f_0' f_1' + f_1 = 0; \quad \eta \to \infty, \quad f_1' = 0 \qquad (7)$$

Note that Eq. (6) is the usual equation for a two-dimensional stagnation point but that the solutions of interest here will involve boundary conditions at $\eta = 0$ given by matching requirements.

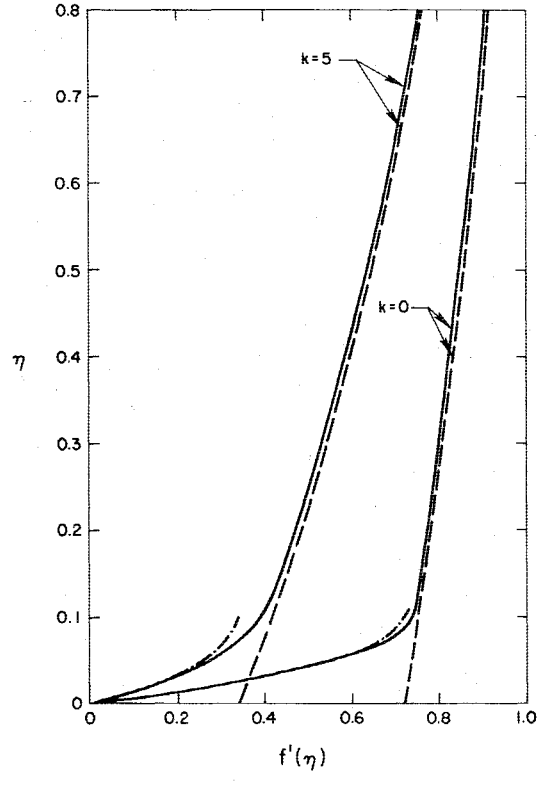

Fig. 1 A comparison of exact numerical and approximate velocity profiles for $f_w = -0.5$ and $\epsilon = 0.1$; —$f'(\eta)$, – – –$f_0'(\eta)$, – · – · –$\hat{f}_1'(\hat{\eta} = \eta/\epsilon)$.

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